

# Kinematics of Galactic Disk

Jozef Klačka

Department of Astronomy and Astrophysics, Faculty for Mathematics and Physics,  
Comenius University, Mlynská dolina, 842 15 Bratislava, Slovak Republic, E-mail:  
klacka@fmph.uniba.sk

**Abstract.** The method of constructing of rotation curve for our galactic disk is developed. Simple physical model shows that the velocity of the Sun around the center of the Galaxy must be less than currently accepted IAU value 220 km/s. Independently obtained result for neutral hydrogen ( $\lambda = 21.1$  cm) confirms the previous statement. The values of the Sun's velocity is about 110 km/s. As a consequence, the rotation curve for the disk of our Galaxy is not flat, but it is a decreasing function of the distance from the center of the Galaxy. This conclusion seems to be consistent also with other galaxies.

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## 1. Introduction

One of the key arguments for the existence of dark matter in the Universe is the existence of flat rotation curves in galactic disks. The existence of such a flat rotation curves is accepted for several decades. As for some references on this theme, we prefer to mention some of the standard textbooks used in the world than an immense number of articles (the greatest part of which the author has either way not in disposal); thus: Mihalas and McRae Routly (1968), Vorontsov-Velyaminov (1978), Mihalas and Binney (1981), Kulikovskij (1985), Scheffler and Elsässer (1988), Zeilik (1994).

## 2. Radial Velocity Data and Trajectories

Let us consider the following simple model (it can be an approximation to real situation if we take into account that mass density exponentially decreases with distance from the center of the Galaxy).

Let us suppose that the mass of the Galaxy is distributed in the form that objects within a few kpc around the Sun move in Keplerian orbits. (Data in Fernie and Hube (1968) show that stars in galactic disk move in strongly noncircular orbits.) We suppose that all objects of the galactic disk move in equatorial plane of the Galaxy, for simplicity. Thus, we consider that a star and the Sun move in galactic plane on Keplerian orbits characterized by the following sets of quantities (their physical interpretation is standard):  $\{p, e, \phi, \omega, R = p/[1 + e \cos(\phi - \omega)]\}$  for the star, and  $\{p_o, e_o, \phi_o, R_o = p_o/[1 + e_o \cos \phi_o]\}$  for the Sun. Simple plane geometry yields  $\sin(\phi - \phi_o) = (r \sin l)/R$ , where  $r$  is distance between the Sun and the star,  $l$  is galactic longitude of the star. Keplerian motion yields for radial velocity of the star with respect to the Sun:

$$\begin{aligned} \frac{\Delta v_r}{\sqrt{\mu}} = & \frac{1}{\sqrt{p}} \left\{ \frac{R_o}{R} \sin l - e [\sin(\phi_o - \omega) \cos l - \cos(\phi_o - \omega) \sin l] \right\} - \\ & - \frac{1}{\sqrt{p_o}} \{ (1 + e_o \cos \phi_o) \sin l - (e_o \sin \phi_o) \cos l \} , \end{aligned} \quad (1)$$

where also  $R = \sqrt{R_o^2 + r^2 - 2 r R_o \cos l}$  may be used.

Making Taylor expansion in  $r/R_o$  (and to the order  $e^2$ ) one can receive

$$\begin{aligned} \frac{\Delta v_r}{\sqrt{\mu/R_o}} = & \left[ 1 - \sqrt{\frac{\mu}{p_o}} (1 + e_o \cos \phi_o) / \sqrt{\frac{\mu}{R_o}} + \frac{e}{2} \cos \alpha - \frac{e^2}{16} (1 + \cos 2\alpha) \right] \sin l \\ & + \left[ \sqrt{\frac{\mu}{p_o}} (e_o \sin \phi_o) / \sqrt{\frac{\mu}{R_o}} - e \sin \alpha + \frac{e^2}{4} \sin 2\alpha \right] \cos l + \\ & + \frac{r}{R_o} \left\{ \frac{1}{4} \left[ 3 - \frac{e}{2} \cos \alpha - \frac{e^2}{8} (9 + \cos 2\alpha) \right] \sin 2l - \right. \\ & \left. - \frac{e}{2} \left( \sin \alpha - \frac{e}{4} \sin 2\alpha \right) \cos 2l \right\} + O \left\{ (r/R_o)^2 \right\} , \end{aligned} \quad (2)$$

where  $\alpha = \phi_o - \omega$ . The quantity  $\mu \equiv G M_{GC}$  is given by mass of the galactic center and its surrounding; circular velocity of the Sun would be  $v_o = \sqrt{\mu/R_o}$ .

In practice we make measurements of  $\Delta v_r$  (Doppler effect) and we do not know values of  $e, \omega$  for individual stars. Thus, we make some assumptions. At first, in order to obtain relations analogous to those which are standardly used, we make averaging of Eq. (2) in  $\omega$  – supposed to be independent on other parameters; thus, using, e. g., (it is supposed that orbits are randomly oriented)  $\langle \sin \omega \rangle = \langle \cos \omega \rangle = 0$ ,  $\langle \sin^2 \omega \rangle = \langle \cos^2 \omega \rangle =$

$1/2, \langle e^k f(\omega) \rangle = \langle e^k \rangle \langle f(\omega) \rangle$  – all these relations correspond to the fact that we neglect all sums containing odd powers of sines and cosines of  $\omega$  – one receives

$$\Delta v_r = \kappa_1 \cos l + \kappa_2 \sin l + A r \sin 2l + r^2 \{a_1 \sin l + a_3 \sin 3l\} , \quad (3)$$

where

$$\begin{aligned} \kappa_1 &= \sqrt{\mu/p_o} e_o \sin \phi_o , \\ \kappa_2 &= \sqrt{\mu/R_o} (1 - (1/16) \langle e^2 \rangle) - \sqrt{\mu/p_o} (1 + e_o \cos \phi_o) , \\ A &= (3/4) \sqrt{\mu/R_o^3} (1 - (3/8) \langle e^2 \rangle) , \\ a_1 &= - (3/32) \sqrt{\mu/R_o^5} (1 - (9/4) \langle e^2 \rangle) , \\ a_3 &= + (3/32) \sqrt{\mu/R_o^5} (7 - (3/4) \langle e^2 \rangle) , \end{aligned} \quad (4)$$

where  $\langle e^2 \rangle$  denotes average value for large number of stars; also higher order in  $r/R_o$  was written in Eq. (3) in comparison with Eq. (2).

### 3. Solar velocity

The important quantity in determining of rotation curve is the velocity of the Sun – velocity of the motion of the Sun around the galactic center.

Standard procedure corresponds to the following situation: We take into account Eq. (3). Now, we will consider only terms with  $\kappa$ -s:

$$\begin{aligned} \kappa_1 &= \sqrt{\mu/R_o} (e_o \sin \phi_o) (1 - (1/2) e_o \cos \phi_o) , \\ \kappa_2 &= - \sqrt{\mu/R_o} \{ (e_o/2) (\cos \phi_o) (1 - (1/4) e_o \cos \phi_o) + (1/16) \langle e^2 \rangle \} . \end{aligned} \quad (5)$$

Now, the least square method reads:

$$\sum \{ \Delta v_{ri} - \kappa_1 \cos l_i - \kappa_2 \sin l_i \}^2 = \min . \quad (6)$$

Differentiation with respect to  $v_o \equiv \sqrt{\mu/R_o}$  and putting the result equal to zero, yields finally

$$v_o = X/Y , \quad (7)$$

where

$$\begin{aligned} X &= (e_o \sin \phi_o) (1 - (1/2) e_o \cos \phi_o) \sum (\Delta v_{ri} \cos l_i) - \\ &- \{ (e_o/2) (\cos \phi_o) (1 - (1/4) e_o \cos \phi_o) + (1/16) \langle e^2 \rangle \} \sum (\Delta v_{ri} \sin l_i) , \end{aligned} \quad (8)$$

$$Y = (e_o \sin \phi_o)^2 \sum \cos^2 l_i + ((1/2) e_o \cos \phi_o)^2 \sum \sin^2 l_i - e_o^2 \sin \phi_o \cos \phi_o \sum \sin l_i \cos l_i . \quad (9)$$

However, correct procedure must use Eq. (2). Again, neglecting terms proportional to  $r/R_o$  we may write

$$\begin{aligned} \frac{\Delta v_r}{\sqrt{\mu/R_o}} &= L_1 \sin l - L_2 \cos l , \\ L_1 &= (e/2) \cos(\phi_o - \omega) \{1 - (e/4) \cos(\phi_o - \omega)\} - \\ &\quad - (e_o/2) \cos \phi_o (1 - (1/4) e_o \cos \phi_o) , \\ L_2 &= e \sin(\phi_o - \omega) \{1 - (e/2) \cos(\phi_o - \omega)\} - \\ &\quad - e_o \sin \phi_o (1 - (1/2) e_o \cos \phi_o) . \end{aligned} \quad (10)$$

Now, the least square method reads:

$$\sum \{\Delta v_{ri} - v_o(L_{1i} \sin l_i - L_{2i} \cos l_i)\}^2 = \min . \quad (11)$$

Differentiation with respect to  $v_o \equiv \sqrt{\mu/R_o}$  and putting the result equal to zero, yields finally ( $\langle \sin \omega \rangle = \langle \cos \omega \rangle = 0$ ,  $\langle \sin^2 \omega \rangle = \langle \cos^2 \omega \rangle = 1/2$ ;  $\langle e^2 \rangle$  denotes average value for large number of stars) the following approximation:

$$v_o = X/Z , \quad (12)$$

where  $X$  is given by Eq. (8) and  $Z$  is given by the following equation:

$$\begin{aligned} Z &= \{(1/2) \langle e^2 \rangle + (e_o \sin \phi_o)^2\} \sum \cos^2 l_i + \\ &\quad + \{(1/8) \langle e^2 \rangle + ((1/2) e_o \cos \phi_o)^2\} \sum \sin^2 l_i - \\ &\quad - e_o^2 \sin \phi_o \cos \phi_o \sum \sin l_i \cos l_i . \end{aligned} \quad (13)$$

Comparison of Eq. (7) and (12) yields that solar velocity must be in reality less than it is supposed up to now. ( $\langle e^2 \rangle \approx e_o^2$  and standard values of  $\kappa$ -s and  $A$  yields that terms with  $\langle e^2 \rangle$  in Eq. (13) are not negligible (!) and one must expect that solar velocity is in several tens of percent less than IAU standard.) Of course, rigorous procedure for the model must use Eq. (2) with the least square method for parameters  $p_o$ ,  $e_o$ ,  $\phi_o$ .

#### 4. Neutral Hydrogen and Solar Velocity

Neutral hydrogen moves in galactic plane practically on circular orbits (see Fig. 8-11, p. 491, Mihalas and Binney 1981). Thus, as a very good approximation we can use for the measured radial velocity  $\Delta v_r$

$$\Delta v_r = (\omega - \omega_o) R_o \sin l , \quad (14)$$

where  $\omega \equiv \omega(R) = v(R)/R$  is angular velocity (in this section) of the gas cloud in distance  $R$  from the galactic center,  $\omega_o = v_o/R_o$  is the analogous quantity for the Sun (circular orbit of radius  $R_o$ ),  $l$  is galactic longitude of the cloud (see, e. g., Eq. (13) in Klačka 1997, or also arbitrary textbook on galactic astronomy already mentioned).

Looking in a direction  $l \in (0^\circ, 90^\circ)$ ,  $\Delta v_r$  (and  $\omega$ ) reaches its maximum value for  $R = R_o \sin l$  (p. 471, Mihalas and Binney 1981). So, we can construct rotation curve for the galactic disk:

$$v(R) = \Delta v_r + v_o \sin l , \quad R = R_o \sin l . \quad (15)$$

Fig. 8-11 (p. 491, Mihalas and Binney 1981) yields, approximately,

$$\Delta v_r [km/s] = 180 - 2 l [^\circ] , \quad (16)$$

for  $l \in (\approx 30^\circ, \approx 70^\circ)$ .

If we use IAU standard value  $v_o = 220$  km/s, Eqs. (15) and (16) enable to construct rotation curve for our Galaxy. We receive standard flat rotation curve in this way.

However, the consequence of the preceding section is that the solar velocity  $v_o$  must be less than 220 km/s. Is it possible to obtain an approximation of real value of  $v_o$  on the basis of neutral hydrogen only? It seems to us that the answer is “yes”. What is the physical interpretation of negative values of  $\Delta v_r$  for  $l \in (\approx 30^\circ, \approx 150^\circ)$  in Fig. 8-11 (p. 491, Mihalas and Binney 1981)? Its minimum value can be approximated by relation

$$\Delta v_r [km/s] = -110 \sin l . \quad (17)$$

Putting this into Eq. (14), one obtains

$$v(R) \frac{R_o}{R} = v_o - 110 km/s . \quad (18)$$

Eq. (18) suggests that we must admit that Eq. (17) physically corresponds to the solar radial velocity – radial velocity of a gas cloud which moves with negligible velocity around the galactic center. Really, if we consider that radius of the Galaxy in neutral hydrogen is analogous to that for other spiral galaxies (see, e. g., Fig. 22, p. 88 in Vorontsov-Velyaminov 1978),  $R_o/R < 1/5$  (approximately) and the left-hand side of Eq. (18) is much less than 110 km/s. If we neglect the left-hand side of Eq. (18), then we obtain

$$v_o \approx 110 km/s . \quad (19)$$

Using this value (or, a little higher), we can again use Eqs. (15) and (16) for constructing rotation curve for our Galaxy. The result is that  $v(R)$  is a decreasing function of  $R$  even in the inner part of the solar orbit ( $R/R_o > 1/2$ , approximately)! If this is true, another important consequence is that dark matter cannot exist within the gas radius of the Galaxy ( $\approx 50$  kpc).

#### 4.1. A Short Comment

This subsection concerns a short comment on the maximum radial velocity along the line of sight at  $l$ .

The important physical relation is given by Eq. (14) and by the relation  $R = R_o \sin l$ . If we want to make Taylor expansion around the value  $R_o$ , we have to make Taylor expansion of Eq. (14), and, finally, we may put  $R = R_o \sin l$  into the obtained result. Thus,

$$\Delta v_r = 2 A R_o (1 - \sin l) \sin l + \frac{1}{2} \omega_o'' R_o^3 (1 - \sin l)^2 \sin l + \dots, \quad (20)$$

where  $A = -\omega_o' R_o / 2$ . Since  $\omega = v(R)/R$ , it can be easily verified that

$$\omega_o'' = v_o'' / R_o + 4 A / R_o^2. \quad (21)$$

Putting Eq. (21) into Eq. (20), one finally obtains

$$\begin{aligned} \Delta v_r = & 2 A R_o (2 - \sin l) (1 - \sin l) \sin l + \\ & + \frac{1}{2} v_o'' R_o^2 (1 - \sin l)^2 \sin l + \dots \end{aligned} \quad (22)$$

This result differs from Eq. (8-26) in Mihalas and Binney (1981; p. 476): they are incorrect since they make Taylor expansion of our Eq. (15) instead of Eq. (14).

## 5. Rotation Curves of Other Galaxies

“The real velocity of rotation equals  $V = V_r \operatorname{cosec} i$ , where  $V_r$  – observed velocity along the line of sight at a distance  $r$  along the observable long axis of a galaxy.” (Vorontsov-Velyaminov 1978, p. 85 – translation from Russian)

The consequence of such a definition is that we must receive flat rotation curves even for Keplerian motion! We will show it. We will consider  $\operatorname{cosec} i = 1$ .

Keplerian motion of a star around the center of mass (a galaxy) is described by the following equations:

$$\begin{aligned} \mathbf{v} &= v_R \mathbf{e}_R + v_T \mathbf{e}_T \\ v_R &= \sqrt{\mu/p} e \sin(\phi - \omega), \quad v_T = \sqrt{\mu/p} [1 + e \cos(\phi - \omega)]. \end{aligned} \quad (23)$$

The fixed coordinate system is defined by the relations

$$\begin{aligned} \mathbf{e}_R &= + \cos \phi \, \mathbf{i} + \sin \phi \, \mathbf{j} , \\ \mathbf{e}_T &= - \sin \phi \, \mathbf{i} + \cos \phi \, \mathbf{j} . \end{aligned} \quad (24)$$

Eqs.(23)-(24) yield

$$\begin{aligned} \mathbf{v} \cdot \mathbf{i} &= v_R \cos \phi - v_T \sin \phi , \\ \mathbf{v} \cdot \mathbf{j} &= v_R \sin \phi + v_T \cos \phi . \end{aligned} \quad (25)$$

As the observed velocity along the line of sight, for one object, we take

$$\mathbf{v} \cdot \mathbf{j} = \sqrt{\mu/p} (\cos \phi + e \cos \omega) , \quad (26)$$

where Eq. (23) was used.

Now, we can put equation  $p = R (1 + e \cos(\phi - \omega))$  into Eq. (26). Making averaging over  $\omega$  and neglecting higher orders of eccentricities, one finally receives

$$\langle \mathbf{v} \cdot \mathbf{j} \rangle_\omega = \sqrt{\mu/R} (1 - e^2/16) \cos \phi . \quad (27)$$

If we make observations at a distance  $r$  along the observable long axis of a galaxy, then  $r = R \cos \phi$  (more correctly, there should be absolute value of  $\cos \phi$ ), and,  $\cos \phi \in (r/R_G, 1)$ , where  $R_G$  is radius of the galaxy. Thus, Eq. (27) yields

$$\langle \mathbf{v} \cdot \mathbf{j} \rangle_\omega = \sqrt{\mu/r} (1 - e^2/16) (\cos \phi)^{3/2} . \quad (28)$$

Since objects of various  $\phi$  occurs at a distance  $r$  along the observable long axis of a galaxy ( $\cos \phi \in (r/R_G, 1)$ ), we take as  $V_r$  the quantity  $\langle \langle \mathbf{v} \cdot \mathbf{j} \rangle_\omega \rangle_\phi$ :

$$V_r = \sqrt{\frac{\mu}{r}} \left(1 - \frac{e^2}{16}\right) \left\{ \sqrt{\frac{r}{R_G}} \arccos\left(\frac{r}{R_G}\right) \right\}^{-1} \int_0^{\arccos(r/R_G)} (\cos \phi)^{3/2} d\phi . \quad (29)$$

It can be easily verified that the function  $V_r(r)$  represents a flat rotation curve.

## 6. Conclusions

We have shown the difference between the standard procedures used in galactic astronomy and the correct procedure for obtaining the velocity of the Sun around the center of the Galaxy. The important result is that real velocity must be less than the IAU standard. Calculations for real data are in preparation.

Independent method based on neutral hydrogen data for our Galaxy confirms the result. Moreover, it yields value of about 110 km/s. As a consequence, the rotation curve

for our Galaxy is not flat (!) and the dark matter cannot exist within the gas radius of the Galaxy ( $\approx 50$  kpc).

Calculations for other galaxies seems to be also consistent with decreasing rotation curve.

We have shown that characterization of rotation curves as flat curves may be the consequence of incorrect physical and mathematical interpretation of the observed data. (The author would like to thank to students for their patient listening of the author's lectures on these themes (also on coordinate systems and transformations of proper motions and on simple kinematics of galactic rotation) during the last year.)

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